Welcome

What to Teach in Mathematics Intervention: Perspectives from Mathematics Education Scholarship

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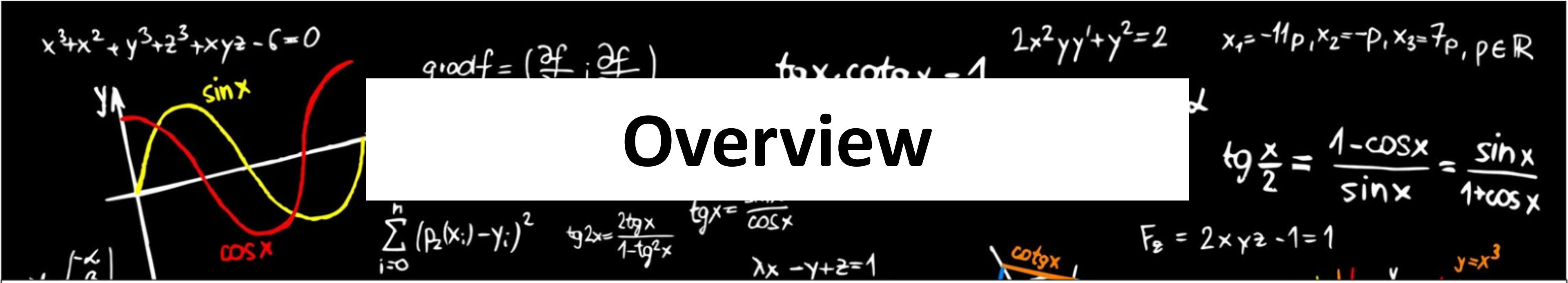
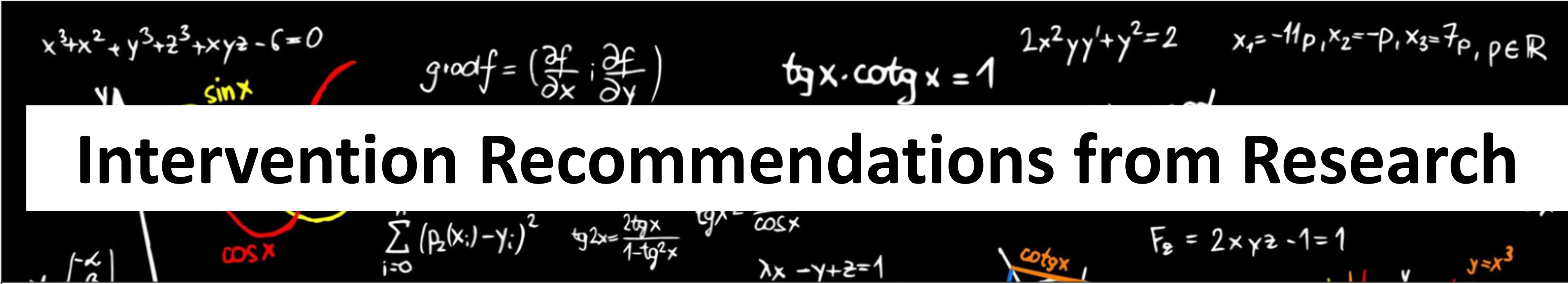


Overview

- What are the important components of effective mathematics interventions?
- Would the teaching of “tricks” be the way to go?
- How can we use intervention sessions to make mathematics memorable and relevant?

What are the Differences?

What if one student had a good understanding of a mathematical concept and the other student had only memorized the idea (or was challenged to effectively memorize the concept or skill)?



Intervention Recommendations from Research

- **C**oncrete--**S**emi-Concrete--**A**bstract (CSA) visual approach
- Explicit instruction
- Underlying mathematical structures

Based on:

Newman-Gonchar, R., Clarke, B., & Gersten, R. (2009). A summary of nine key studies: Multi-tier intervention and response to interventions for students struggling in mathematics. Portsmouth, NH: RMC Research Corporation, Center on Instruction.

Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement.* New York: Routledge.

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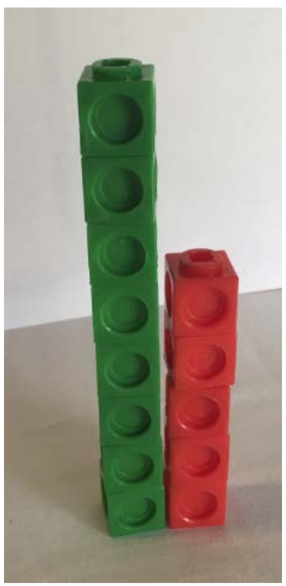
Additive - Compare Problems

- Lucy has 3 fewer apples than Julie. Lucy has 5 apples. How many apples does Julie have?

C



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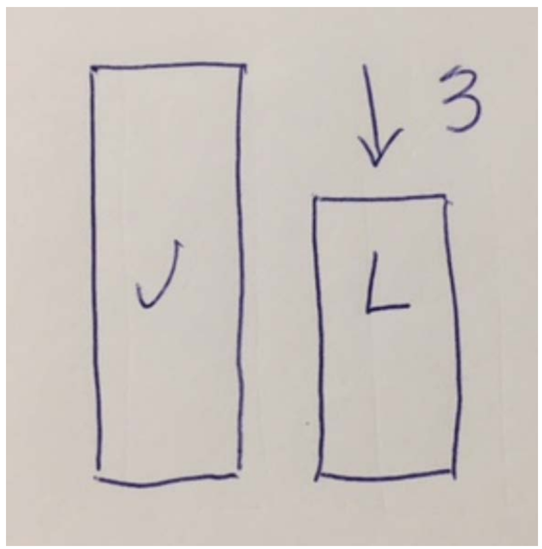

A

$? - 3 = 5$

What Takes Place during Intervention Sessions?

- What do you think has a strong influence on the strategies teachers use during mathematics intervention sessions?

So, what did you learn in elementary school?

- With the person sitting next to or around you, discuss these rules – were you taught them in elementary school?
- Decide if the rules shown at the right are always true.
- If the rule is not always true, find a counterexample.

- Addition makes numbers bigger or multiplication makes numbers bigger.
- When you multiply a number by 10, just put a 0 on the end of the number.
- The longer the number, the larger the number.

Addition and multiplication make “bigger”

32 + 67 = 99

– 3 + (–14) = –17

–17 is neither larger than –3 nor –14.

15 + 0 = 15

15 x 10 = 150

$$\frac{1}{3} \times \frac{2}{7} = \frac{2}{21}$$

0.25 x 0.16 = 0.04

15 x 0 = 0

When you multiply by 10, just put a 0 on the end of the number.

15 x 10 = 150

4.5 x 10 = 45.0

4.5 x 10 ≠ 4.50

The longer the number, the larger the number.

1,278,931 > 1,469

1.3 > 1.0118743

1.02 < 1.2

Impact of Teaching Rules that Expire

- Students use rules as they have interpreted them.
- They often do not think about the rule beyond its immediate application.
- When even the strongest math students find that a “rule” doesn’t work, it is unnerving and scary.

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Goal – Try to AVOID DEAD ENDS

"13 Rules that Expire" (Karp, Bush & Dougherty, August 2014 in *Teaching Children Mathematics*)

Take the Oath!! Nevermore:

- Borrowing
- Carrying
- “Reducing” fractions
- Talking about Fractions as a Top Number and Bottom Number
- “Plugging” numbers into the equation
- Getting “rid” of the decimal

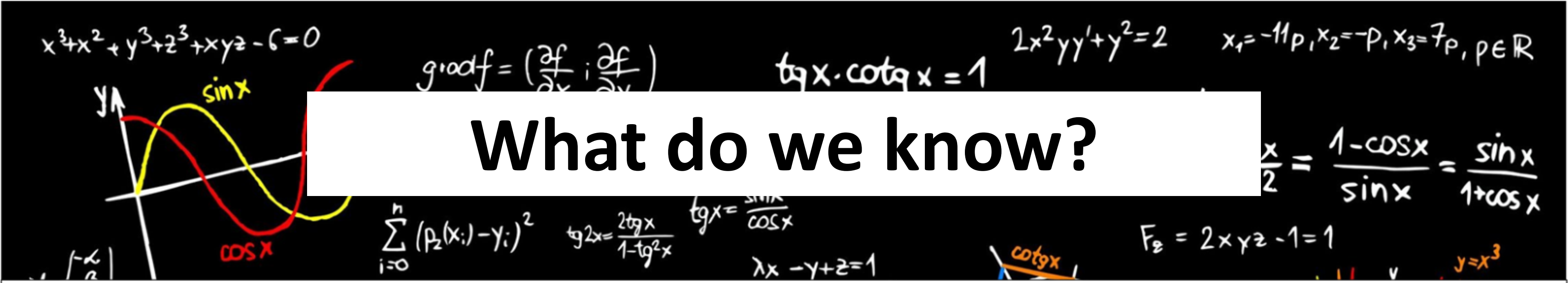
What Evidence do we have from High School?

Mari said, "2*t* is always greater than *t* + 2." Do you agree with Mari?

- A. Yes, because multiplication always gives you a larger answer than addition.
- B. Yes, because *t* is a positive number.
- C. No, because multiplication is not the inverse of addition.
- D. No, because it is possible that 2*t* can be equal to or less than *t* + 2.

41.6% second semester Algebra I students selected A as their answer.

Dougherty, Barbara. J. & Anne Foegen, A.(2014) *Concept of variable: Screening and progress monitoring tool*. Ames, IA: Iowa State University.



What do we know?

- Telling isn't teaching.
- Told isn't taught.
- Interventions provide opportunities to spend time actively developing mathematical structure.

Active Mathematics:
Boaler, J. & Selling, S.K. (2017) Psychological imprisonment or intellectual freedom? A longitudinal study of contrasting school mathematics approaches and their impact on adults' lives. *Journal for Research in Mathematics Education* 48 (1), 78-105.

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Creating Mental Residue

- Establishing foundational understanding

- Modeling the physical action is the important part and the memory of that action doesn't fade

- Actively engaging and "doing" the process supports students' thinking

Dougherty, B. J. (2008). Measure up: A quantitative view of early algebra. In Kaput, J. J., Carraher, D. W, & Blanton, M. L. (Eds.), *Algebra in the early grades*, (pp. 389–412). Mahwah, NJ: Erlbaum.
Boaler, J. & Selling, S.K. (2017) Psychological imprisonment or intellectual freedom? A longitudinal study of contrasting school mathematics approaches and their impact on adults' lives. *Journal for Research in Mathematics Education* 48 (1), 78-105.

Danger - Key Words Ahead

Mark has 3 packages of pencils. There are 6 pencils in each package. How many pencils does he have <u>in all</u>?

Adapting Reading Strategies to Teach Mathematics

What is the fundamental message the kids get when told to search for the key word?

- Don't read the problem.
- Don't imagine the situation.
- Ignore that context.
- Abandon your prior knowledge.

You don't have to read. You don't have to think. Just grab the numbers and compute.

Hyde. A. A. (2006). *Comprehending math: Adapting reading strategies to teach mathematics, K-6*. Portsmouth, NH: Heinemann.

Why Avoid a Key Word Strategy?

- The use of a Key Word Strategy does not—
  - Develop of sense making or support making meaning
  - Build structures for more advanced learning
  - Appear in many problems

- Students consistently use Key Words inappropriately

- Multi-step problems are impossible to solve with a Key Word Strategy (and two step problems start in 2nd grade)

Clement & Bernhard, 2005 A problem solving alternative to using key words.
Van de Walle, J., Karp, K., & Bay Williams, J. (2016). *Elementary and Middle School Mathematics: Teaching developmentally*. New York: Pearson.

Meanwhile, at the Local Elementary School

Grade	Process	E means	P means	S means	U means
Key Word Posters					
1	CUBES	equation	--	solve	underline important words
2	SUPER	explain with a number sentence	picture	slowly read the problem	underline the question
3	FUSE	explain your thinking	--	select a strategy	understand
4	QTIPS	--	plan	solution	--
5	SWEEP	equation explain	pictures	symbols	--

Andrews, D. & Kobett, B. M. (2017, July). *Connection to discourse: Word problems.* Presentation for the NCTM Discourse Institute, Baltimore, MD.

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Are Students Making Sense of Word Problems?

- Are students largely using the approach of "what have we been doing this week"?
- Are students learning strategies in intervention class to support sense making?

The Infamous Shepherd Problem

There are 25 sheep and 5 dogs in a flock. How old is the shepherd?

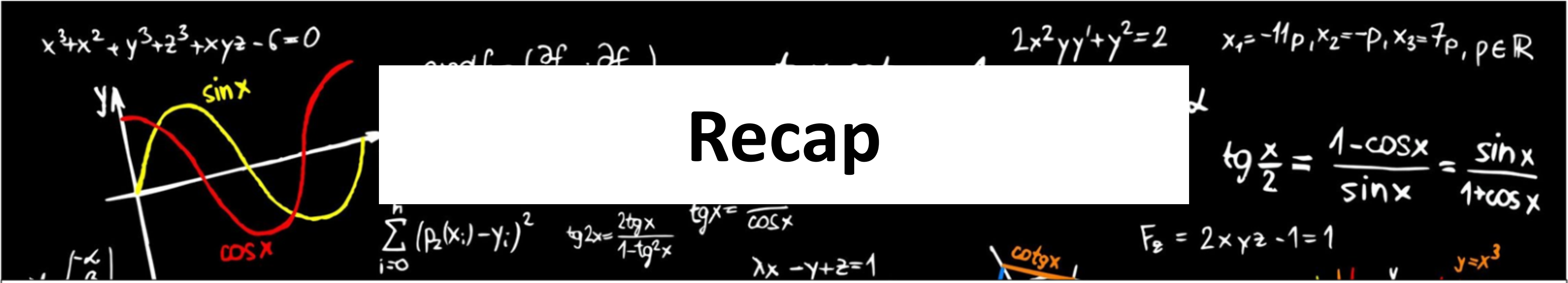
Caldwell, Kobett & Karp (2014) *Addition and Subtraction in Practice*. NCTM.

Can an Intervention Provide time to Discuss Options?

How could students talk about which of the following three options would be the correct answer?

- The shepherd is 30 years old
- The shepherd is 125 years old; and
- It is not possible to tell the shepherd's age from the information given in the problem.

Caldwell, Kobett & Karp (2014) *Essential understanding of addition and subtraction in practice, grades K-2*. NCTM.



Recap

- Avoid giving students "math tricks" and rules without full connections
- Engage students in "doing mathematics" at a pace that allows them to discuss ideas and concepts
- AND…

Are we working as a group or team?

- Teachers in schools?
- Teacher education and school districts?
- Researchers?
- What would it take to shift the work from that of a group to that of a team?
- How would this shift change our goals, the level of cohesiveness in our discussions, and the outcomes of our work on the lives of children?

Donaldson, S. 2017 Coaches Session

What is the Whole School Agreement?

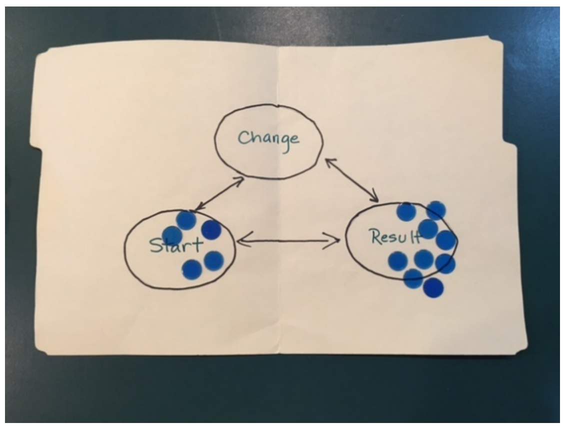
- Decide on the language and models everyone will use – focusing on precision and consistency
- Prepare all students, from the beginning, to walk out of the building with the mathematical knowledge and processes they need
- Engage each and every student in “doing mathematics” to build long lasting understanding

Cai, J. (2010). Helping elementary school students become successful mathematical problem solvers. In D. Lambdin (Ed.), *Teaching and learning mathematics: Translating research to the classroom* (pp. 9–14). Reston, VA: NCTM.
Karp, Bush & Dougherty (2016) *Establishing a Mathematics Whole School Agreement*. NCTM.
Stein, Smith, Henningsen & Silver, 2000 - Mathematical Tasks Framework

Acting as a Team to Agree on Language, Models, and Notation Across the School

- Language ~~Borrowing/Carrying~~ → Regrouping/Trading

- Models



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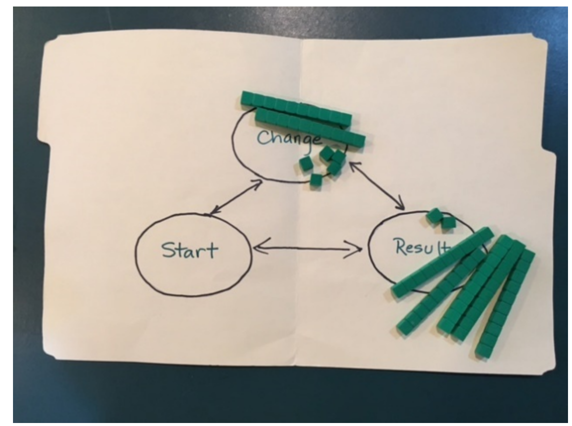
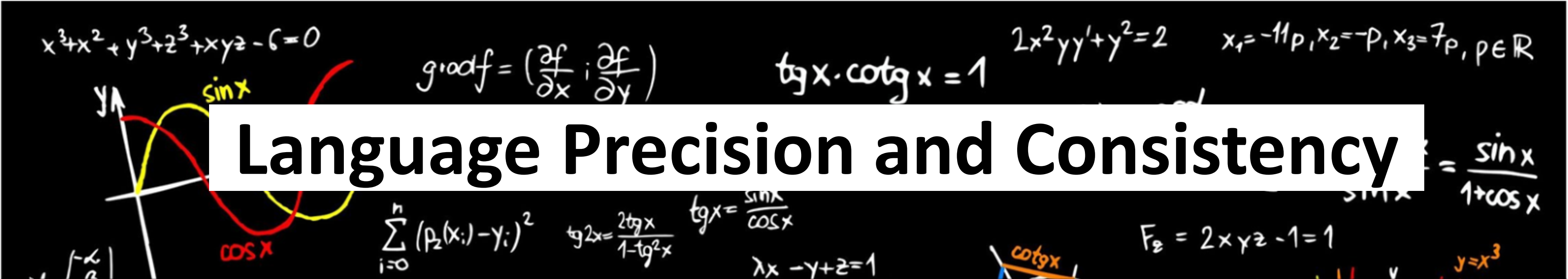


- Notation $4 + 5 = \square$ → $17 + x = 26$ →

using a letter as a variable $3d$

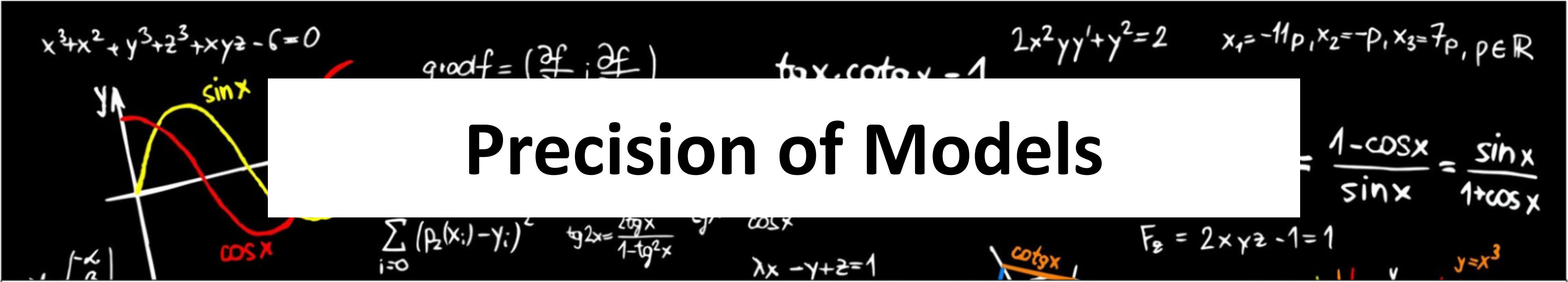
Teachers were Asked

1. What are the models your sch
agree to use?
2. What is the language that you ag
use? What language should be avoid
3. What notations should be used? Must be avoided?
4. What is an example of a concept or model moving vertically up the grades?



Language Precision and Consistency

- Regardless of the grade level, the name of the mathematical property will be used: commutative property of addition, commutative property of multiplication, distributive property of multiplication over addition and so on.
- Avoid: Flip-Flop Property or the Ring Around the Rosie Property for naming the Commutative Property



Precision of Models

- Use a trajectory of number lines – from focusing on the unit in K – to an empty number line
- Use visual Schema models to develop the meaning of word problems
- Use a relational approach to demonstrate the meaning of the equal sign

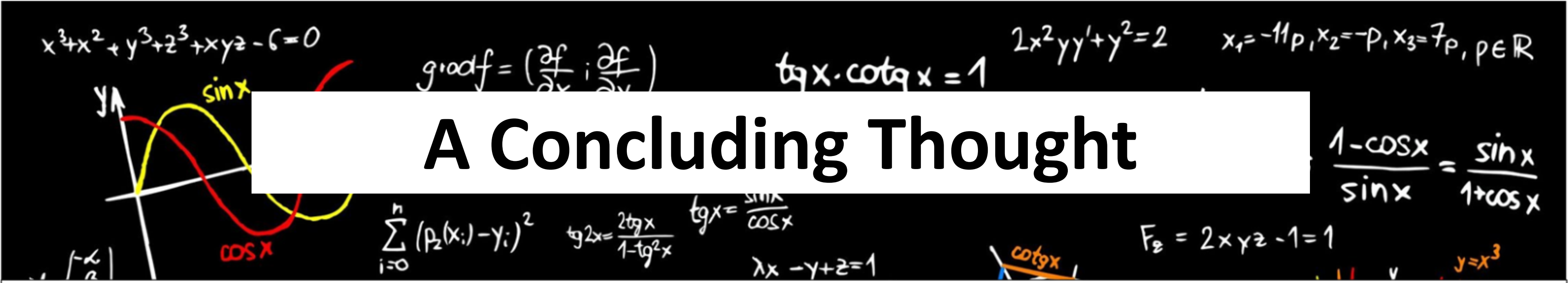
Recap – What Should be Emphasized in Interventions

- Action and the importance of "doing mathematics"

- By carrying out the actions – mental residue results!!

- Use intervention sessions as opportunities to make math MEMORABLE

Mental Residue - Dougherty, B. J. (2008). Measure up: A quantitative view of early algebra. In Kaput, J. J., Carraher, D. W., & Blanton, M. L. (Eds.), *Algebra in the early grades*, (pp. 389–412). Mahwah, NJ: Erlbaum.



A Concluding Thought

We expect that the very best doctors will treat the most grievously ill patients. It should be no different in mathematics education. Great teachers have the skills to help the students who struggle the most. (Larson, 2011)

Larson, M. (2011).. Supporting students' achievement of the Common Core. NCTM Conference presentation. Indianapolis, IN.

Thank you

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