Meta-Analysis Overview and Results

Introduction. Several studies have demonstrated that knowledge and understanding of fractions is predictive of mathematics performance in middle and high school above and beyond intellectual abilities, general mathematics achievement, and socioeconomic status (Siegler et al., 2012). Unfortunately, many students are not adequately learning fractions and other rational numbers topics when they are introduced in the mathematics curriculum. As rational number topics (e.g., fractions, ratios, proportions) become increasingly complex through upper elementary and early middle school, the gap between lower- and higher-performing students tends to widen. Therefore, providing evidence-based rational numbers intervention to students experiencing mathematics difficulties is critical to prevent pernicious, long-term outcomes. There has been a recent outpouring of evidence focused on rational numbers interventions as its importance has gained increasing attention from educators and researchers. To synthesize the empirical research on rational numbers interventions, and to increase confidence in the results, contemporary meta-analytic techniques were used in this study. The results of the meta-analysis provide valuable information to both educators and researchers.

Purpose of the Meta-Analysis. Our goal was to evaluate which aspects of rational numbers interventions were associated with positive student outcomes. We examined which *instructional components* (e.g., visual representations, use of number lines, teaching and using of mathematical language) and *study features* (e.g., group size, interventionist training) may have contributed to the effectiveness of intervention. In choosing to focus on instructional components and study features, the findings have the capacity to inform state and district leaders, as well as teachers providing intervention, on which instructional components are necessary for a rational numbers intervention to be effective.

Inclusion Criteria. We searched for studies that focused on teaching rational numbers concepts to students experiencing mathematics difficulties in Grades 3 through 9. See Figure 1 for a flow diagram detailing the eligibility coding for the 1,424 candidate reports yielded from our initial and extended search procedures. Following title and abstract screening, a majority of reports ($n = 1,347$) were excluded, leaving 77 reports to be screened at the full-text level for final eligibility status. After full-text screening, 51 ineligible reports were excluded, primarily due to: (a) a participant group that did not include a sufficient proportion of students with or at risk for difficulties in mathematics; (b) failure to meet WWC group design standards; (c) lack of relation of intervention content to rational number concepts or operations; (d) ineligibility of research design; or (e) absence of eligible outcomes in the study. A total of 26 studies with 4,237 unique participants from 30 independent samples were deemed eligible for inclusion in the final meta-analysis. These samples provided a total of 115 effect sizes.

Meta-Analysis Results: Across all studies, we found significant mean effects favoring intervention. The estimate of the mean effect size across all 26 studies (115 effect sizes) included in the analysis was 0.65 and differed significantly from zero ($p < .001$, 95% CI [0.50, 0.80]). We used a multivariate meta-regression model with RVE to assess the possible moderating effects of instructional components. As indicated by the mean effect sizes (i.e., the intercepts from the RVE meta-regression model; see Table 1), the majority of the instructional components were non-significant at $p < 0.05$, with the exception of the teaching and use of mathematical language

($p < .047$). Ten studies with 62 effect sizes reflected interventions including the *teaching and using of mathematical language*, indicating that studies including this instructional component were associated with significantly larger positive effects on the outcomes.

Using univariate meta-regression models, we explored 10 study features as categorial moderator variables (see Table 2). We found a significant relationship ($p < 0.005$) between grade level and effect size: interventions for students in elementary grades (3 through 6) had larger effects than those for students in middle school (7 through 9). Interventions delivered to small groups had significantly larger ($p < 0.005$) effects than those delivered in large-group settings. Interventions delivered by research project personnel were significantly more effective ($p < 0.005$) than those delivered by school personnel. We also found that interventions longer than nine hours (i.e., interventions 10–19 hours and interventions 20 hours or longer) were more effective than shorter interventions (0–9 hours). However, only the specific comparison with the category of interventions lasting 20 hours or longer was statistically significant ($p < 0.005$). Finally, interventions for which the interventionists received ongoing training were significantly more effective than those without ($p < 0.005$).

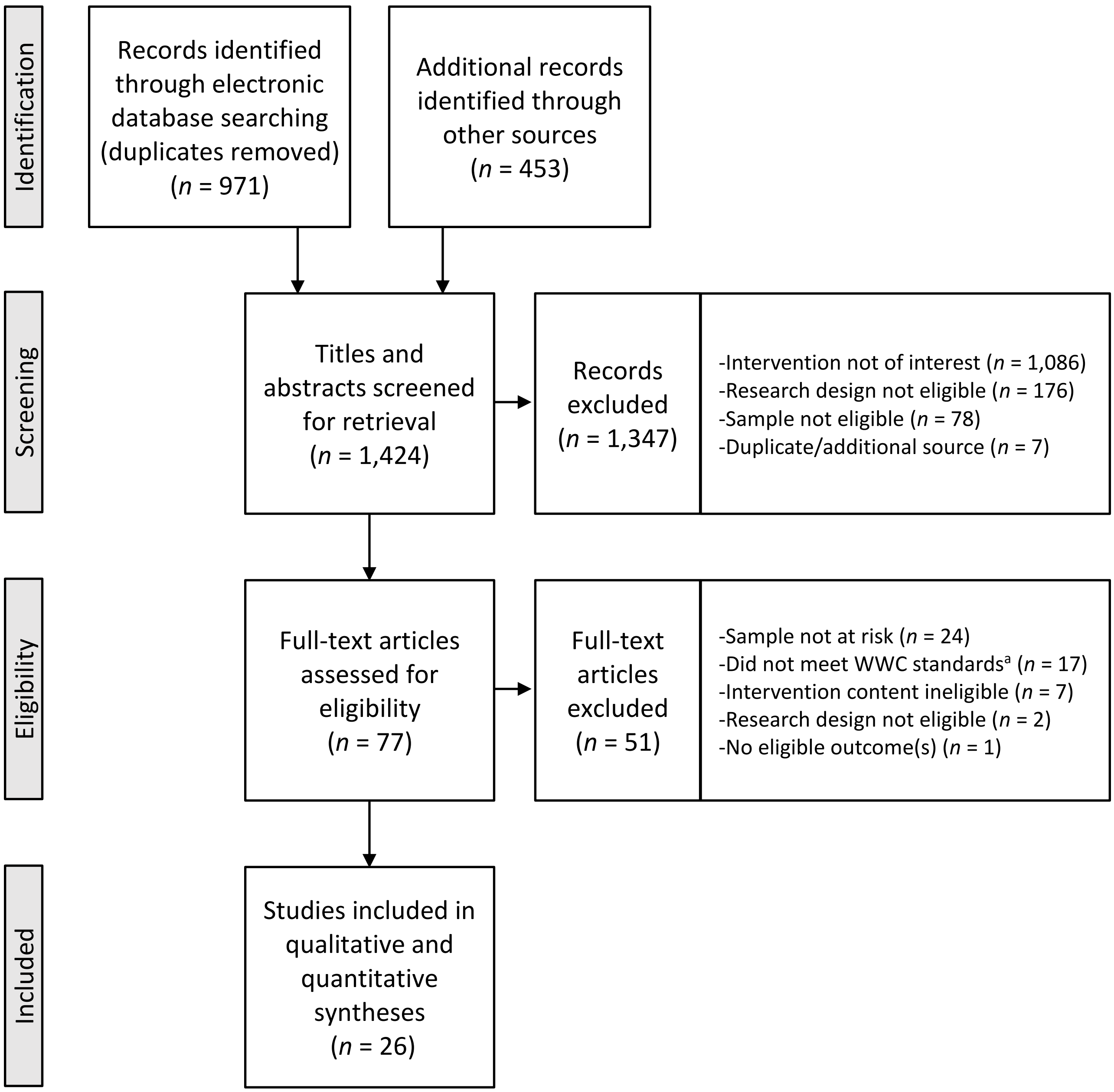
To better understand the role of the number line within rational numbers intervention we explored whether its use explained any variability while also including important controls in the meta-regression model (i.e., group size, grade level, intervention duration, and the nature of the comparison condition). We found no significant relationship ($b = 0.68$, $p = 0.07$, 95% CI [−0.13, 1.48]). However, degrees of freedom were less than four, which likely underestimates the true Type 1 error (Tipton, 2015). A sensitivity analysis examining the meta-regression results without the presence of the control variables yielded significant results ($p < 0.0001$). Whether or not the number line was associated with significant results is, therefore, inconclusive.

Practical Implications: The results from this meta-analysis suggest that intervention programs devoting time to *teaching and using of mathematical language* can substantially enhance outcomes. Mathematical language is a type of abstract academic language—terms such as *equivalent*, *reciprocal, circumference*—that helps students learn mathematics concepts more precisely. When students understand and use mathematical language, it is believed that the students will more deeply understand the mathematics they are learning.

Findings from this study provide valuable information that may help educators understand effective intervention components for students experiencing mathematics difficulties and the conditions under which intervention is optimal. Specifically, the positive impact for *teaching and using of mathematical language* may guide schools and districts in choosing interventions that include this practice. Also, the impact for interventionists receiving high-quality, ongoing training may guide districts when making decisions on how to provide training and professional development to teachers who deliver intervention. Providing evidence-based intervention on rational numbers topics addresses the ever-widening achievement gap in mathematics between low- and high-performing students as they prepare for high-stakes courses like Algebra 1. Findings from this project should be considered as schools make important decisions about the provision of mathematics services to students experiencing difficulties in mathematics.

Figure 1

Study identification flow diagram following PRISMA guidelines



[a] The study is a randomized controlled trial with high attrition or a quasi-experimental design study with analysis groups that are not shown to be equivalent.

Table 1

Relationships Between Instructional Components and Effect Sizes Using Multivariate Meta-Regression Models

Variable	*k*(*n*)	*b* [95% CI]	SE	*df*	*p*	τ^2
Instructional Components						0.00
Explicit	19(101)	−0.34 [−0.87, 0.18]	0.18	3.76[a]	0.142[a]	
Representations	23(108)	−0.38 [−1.73, 0.96]	0.52	4.76	0.491	
Strategic Prompting Tools	14(87)	0.18 [−0.27, 0.63]	0.19	6.88	0.371	
Cumulative Review	14(79)	−0.22 [−0.59, 0.14]	0.15	6.90	0.183	
Teaching and Use of Mathematical Language	10(62)	0.57 [0.01, 1.14]	0.23	6.53	0.047*	
Timed Fluency-Building Activities	10(67)	0.17 [−0.40, 0.74]	0.14	2.13[a]	0.337[a]	
Intervention and Design Characteristics (Controls)						
Group Size (small group, large group)		−0.34 [−1.04, 0.35]	0.28	6.08	0.275	
Grade Level (3rd-6th, 7th-9th)		−0.20 [−1.17, 0.77]	0.39	5.69	0.631	
Duration (≤ 9 hrs, 10-19 hrs, ≥ 20 hrs)		0.09 [−0.16, 0.34]	0.11	8.58	0.445	
Comparison Condition (BAU, active alt. treatment)		0.02 [−0.65, 0.68]	0.22	3.32[a]	0.944[a]	
Number Line Representation						0.00
Number line (with controls)	10(74)	0.68 [–0.13, 1.48]	0.19	1.98[a]	0.069	
Number line (without controls)		0.52 [0.34, 0.69]	0.08	22.89	0.000*	

Note. The assumed average intercorrelation across all variables, rho (ρ), is set at .80. A multivariate meta-regression model (simultaneous model) was estimated using robust variance estimation (RVE) to handle statistically dependent effect sizes. *k* = number of studies; *n* = number of effect sizes; CI = confidence interval.

[a] Degrees of freedom is less than four, thus the *p*-value is untrustworthy due to small sample size (Tipton, 2015).

* $p < .05$.

Table 2

Exploratory Analyses: Relationships Between Study Features and Effect Sizes

Moderator	*k*(*n*)	*b* [95% CI]	SE	df	*p*	τ^2
Type of Measure[a]						0.02
Researcher Developed	21(62)	0.10 [−0.16, 0.35]	0.12	24.27	0.44	
Independent	19(53)	0 (Ref)				
Outcome Domain[a]						0.00
Knowledge of Rational Numbers	16(28)	0 (Ref)				
Rational Numbers Magnitude	11(31)	0.47 [0.22, 0.72]	0.12	16.85	0.00***	
Fractions Computation	5(7)	0.24 [−0.20, 0.69]	0.17	4.32	0.21	
Fractions Addition and Subtraction	12(19)	0.52 [0.14, 0.90]	0.18	17.03	0.01*	
Word Problems	9(20)	−0.02 [−0.39, 0.35]	0.16	9.18	0.89	
Problem Solving	5(8)	0.06 [−0.57, 0.69]	0.25	5.12	0.81	
Grade Level						0.00
Elementary (3^{rd}–6^{th})	12(78)	0.43 [0.22, 0.63]	0.10	22.56	0.00***	
Middle School (7^{th}–9^{th})	14(37)	0 (Ref)				
Group Size						0.00
Large Group (> 6)	9(26)	0 (Ref)				
Small Group (2−6 students)	14(84)	0.47 [0.28, 0.66]	0.09	21.38	0.00***	
Interventionist						0.00
Research Project Personnel	12(78)	0 (Ref)				
School-Based Personnel	9(26)	−0.43 [−0.67, −0.20]	0.11	14.85	0.00***	
Technology Usage						0.00
None	17(92)	0 (Ref)				
Partial or full	9(23)	−0.33 [−0.61, −0.05]	0.13	12.63	0.03*	
Duration						0.00
≤ 9 hours	7(17)	0 (Ref)				
10−19 hours	6(23)	0.41 [−0.28, 1.11]	0.30	8.05	0.21	
≥ 20 hours	13(75)	0.45 [0.24, 0.66]	0.09	8.61	0.00***	
Initial Training						0.02
None	4(8)	0 (Ref)				
< 1 day	5(17)	0.23 [−0.25, 0.70]	0.20	6.83	0.30	
> 1 day	15(85)	0.37 [0.02, 0.72]	0.13	4.19	0.04*	
Ongoing Training						0.00
No	13(32)	0 (Ref)				
Yes	13(83)	0.48 [0.29, 0.67]	0.09	22.27	0.00***	
Instructional Setting						0.00
Special Education	6(19)	0 (Ref)				
General Classroom	5(13)	−0.18 [−0.65, 0.30]	0.20	6.73	0.41	
Supplemental Intervention	15(83)	0.19 [−0.26, 0.64]	0.18	6.14	0.35	

Note. Univariate meta-regression models were estimated for each moderator variables using robust variance estimation (RVE) to handle statistically dependent effect sizes. [a] Meta-

regression models were estimated at the outcome level, not the study level. The variables labeled "(Ref)" are the reference categories to which the other categories are being compared. k = number of studies; n = number of effect sizes; CI = confidence interval.
* $p < .05$. ** $p < .01$. *** $p < .005$ (the Bonferroni corrected critical p value).