Meta-Analysis Overview and Results

Introduction. Several studies have demonstrated that knowledge and understanding of fractions is predictive of mathematics performance in middle and high school above and beyond intellectual abilities, general mathematics achievement, and socioeconomic status (Siegler et al., 2012). Unfortunately, many students are not adequately learning fractions and other rational numbers topics when they are introduced in the mathematics curriculum. As rational number topics (e.g., fractions, ratios, proportions) become increasingly complex through upper elementary and early middle school, the gap between lower- and higher-performing students tends to widen. Therefore, providing evidence-based rational numbers intervention to students experiencing mathematics difficulties is critical to prevent adverse long-term impacts. There has been a recent outpouring of evidence focused on rational numbers interventions as their importance gained increasing attention from educators and researchers. To synthesize the empirical research on rational numbers interventions, and to increase confidence in the results, contemporary meta-analytic techniques were used in this study. The results of the meta-analysis provide valuable information to both educators and researchers.

Purpose of the Meta-Analysis. Our goal was to evaluate which aspects of rational numbers interventions were associated with positive student outcomes. We examined which instructional components, intervention characteristics, and study design features may have contributed to the effectiveness of intervention. The findings have the capacity to inform state and district leaders, as well as teachers providing intervention, on which instructional components and other factors are necessary for a rational numbers intervention to be effective.

Inclusion Criteria. We searched for studies that focused on teaching rational numbers concepts to students experiencing mathematics difficulties in Grades 3 through 9. See Figure 1 for a flow diagram detailing the eligibility coding for the 1,654 candidate reports yielded from our initial and extended search procedures. Following title and abstract screening, 1,553 reports were excluded, leaving 101 reports to be screened at the full-text level for final eligibility status. After full-text screening, 52 reports were excluded, primarily due to: (a) a participant group that did not include a sufficient proportion of students with or at risk for difficulties in mathematics; (b) a lack of relation of intervention content to rational number concepts or operations; (c) ineligibility of research design; or (d) absence of eligible outcomes in the study. An additional 21 studies did not meet WWC standards and were excluded after the quality appraisal stage. A total of 28 studies with 3,853 unique participants. These samples provided a total of 90 effect sizes.

Meta-Analysis Results. Across all studies, we found significant mean effects favoring intervention. The estimate of the mean effect size across all 28 studies (90 effect sizes) included in the analysis was 0.60 and differed significantly from zero ($SE = 0.04$, $p < .001$, 95% CI [0.51, 0.85]). We used a multivariate meta-regression model with robust variance estimation (RVE) to assess the possible moderating effects of instructional components. As indicated by the mean effect sizes (i.e., the intercepts from the RVE meta-regression model; see Table 1), most of the instructional components were non-significant at $p < .05$, except for the teaching and use of mathematical language ($p < .04$).

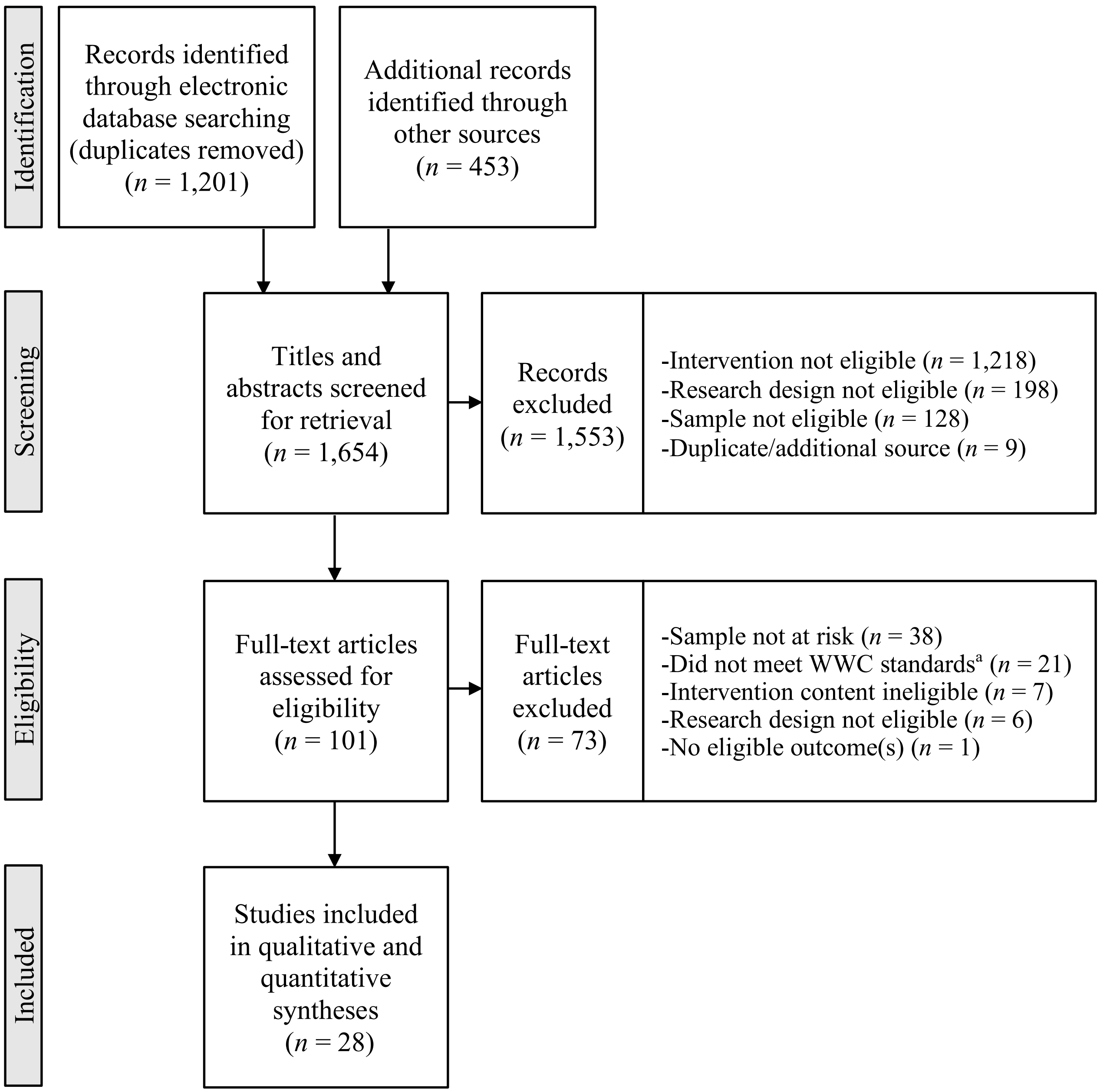
Using univariate meta-regression models, we explored intervention characteristics and study features as moderator variables (see Table 2). We found a significant relationship (*p* less than a Bonferroni-corrected critical *p* value of .0045) between grade level and effect size: interventions for students in elementary grades (3 through 6) had larger effects than those for students in middle school (7 through 9). Interventions delivered to small groups had significantly larger effects than those delivered in large-group settings. Interventions delivered by research project personnel were significantly more effective than those delivered by school personnel. We also found that interventions longer than nine hours (i.e., interventions 10–19 hours and interventions 20 hours or longer) were more effective than shorter interventions (0–9 hours). However, only the specific comparison with the category of interventions lasting 20 hours or longer was statistically significant. Finally, interventions for which the interventionists received ongoing training were significantly more effective than those without.

Practical Implications. The results from this meta-analysis suggest that intervention programs devoting time to teaching and use of mathematical language can substantially enhance outcomes. Mathematical language is a type of abstract academic language—terms such as equivalent, reciprocal, circumference—that helps students learn mathematics concepts more precisely. When students understand and use mathematical language, it is believed that the students will more deeply understand the mathematics they are learning.

Findings from this study provide valuable information that may help educators understand effective intervention components for students experiencing mathematics difficulties and the conditions under which intervention is optimal. Specifically, the positive impact for teaching and using of mathematical language may guide schools and districts in choosing interventions that include this practice. Also, the impact for interventionists receiving high-quality, ongoing training may guide districts when making decisions on how to provide training and professional development to teachers who deliver intervention. Providing evidence-based intervention on rational numbers topics addresses the ever-widening achievement gap in mathematics between low- and high-performing students as they prepare for high-stakes courses like Algebra 1. Findings from this project should be considered as schools make important decisions about the provision of mathematics services to students experiencing difficulties in mathematics.

Figure 1

Study Identification Flow Diagram Following PRISMA Guidelines



[a] These studies were randomized controlled trials with high attrition or a quasi-experimental design studies with analysis groups that were not shown to be equivalent.

Table 1

Relationships Between Instructional Components and Effect Sizes Using Multivariate Meta-Regression Models

Variable	*k*(*n*)	*b* [95% CI]	SE	*df*	*p*	τ^2
Instructional components						0.25
Fluency activities	10(46)	0.34 [−0.67, 1.35]	0.32	3.02	0.36[a]	
Mathematical language	12(51)	0.50 [0.03, 0.98]	0.18	4.77	0.04*	
Review	18(65)	−0.48 [−1.04, 0.07]	0.23	5.90	0.08	
Student explanations	11(49)	−0.02 [−1.48, 1.44]	0.51	3.72	0.97[a]	
Visual representations	20(74)	−0.11 [−0.67, 0.46]	0.23	6.06	0.66	

Note. Three variables were included in this model as controls: group size (small group, large group), duration (≤ 9 hrs, 10–19 hrs, ≥ 20 hrs), and comparison condition (BAU, active alt. treatment). *k* = number of studies; *n* = number of effect sizes.

[a] Degrees of freedom are less than four, which may yield an inaccurate p-value.

* $p < .05$.

Table 2

Relationships Between Intervention Characteristics or Study Design Features and Effect Sizes

Intervention characteristics	*k*(*n*)	*b* [95% CI]	SE	df	*p*	τ^2
Grade level						0.19
Elementary (3–6)	14(57)	0.49 [0.23, 0.75]	0.13	25.14	0.00***	
Middle school (7–9)	14(33)	0 (Ref)				
Group size						0.21
Large group (> 6)	9(23)	0 (Ref)				
Small group (2–6 students)	17(63)	0.49 [0.19, 0.79]	0.14	16.79	0.00***	
Interventionist						0.21
Research project personnel	15(58)	0.48 [0.17, 0.78]	0.14	17.06	0.00***	
School-based personnel	9(23)	0 (Ref)				
Duration of intervention						0.30
≤ 9 hours	7(16)	0 (Ref)				
10−19 hours	6(14)	0.58 [0.01, 1.16]	0.26	10.47	0.05*	
≥ 20 hours	15(60)	0.49 [0.24, 0.73]	0.11	10.88	0.00***	
Initial training						0.29
None or not reported	5(11)	0 (Ref)				
< 1 day	6(13)	0.05 [−0.49, 0.58]	0.24	8.68	0.85	
> 1 day	17(65)	0.35 [−0.05, 0.77]	0.17	6.17	0.07	
Ongoing training						0.19
No	13(30)	0 (Ref)				
Yes	15(60)	0.52 [0.25, 0.78]	0.13	24.74	0.00***	
Instructional setting						0.26
Special education	8(22)	0 (Ref)				
General education classroom	5(10)	−0.27 [−0.75, 0.21]	0.21	9.00	0.24	
Supplemental intervention	15(58)	0.18 [−0.27, 0.62]	0.21	13.21	0.41	
Intervention content						0.25
Decimals only	2(4)	−0.41 [−1.55, 0.73]	0.14	1.25	0.17[b]	
Fractions only	19(65)	0 (Ref)				
Multiple content topics	7(21)	−0.12 [−0.56, 0.32]	0.20	10.81	0.55	
Content level						0.29
Foundational	6(14)	−0.31 [−0.69, 0.08]	0.17	8.46	0.10	
Grade level	6(13)	−0.23 [−0.76, 0.31]	0.23	8.34	0.36	
Study design features	***k*(*n*)**	***b* [95% CI]**	**SE**	**df**	***p***	**τ^2**
Comparison condition						0.28
Business as usual	23(78)	0 (Ref)				
Aligned alternative treatment	5(12)	−0.50 [−0.76, −0.23]	0.11	5.63	0.00***	
Type of measure[a]						0.29
Researcher developed	23(47)	0.17 [−0.11, 0.44]	0.13	26.21	0.22	
Independent	22(43)	0 (Ref)				

Note. Study counts for group size and interventionist excluded the two studies for which the intervention was entirely conducted on a computer. Ref = reference category to which others are compared; *k* = number of studies; *n* = number of effect sizes; *b* = unstandardized coefficient.
[a] Meta-regression model was estimated at the outcome level, not the study level.
[b] Degrees of freedom are less than four, which may yield an inaccurate p-value.
* $p < .05$. ** $p < .01$. *** $p < .0045$ (the Bonferroni corrected critical p value).