Improving Student Mathematics Performance in Three Distinct Middle Schools:

Technical Report of Year 1 Implementation

2022

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This project was funded by the U.S. Department of Education (Grant No. DRL-H326M170003).

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Jayanthi, M., Schumacher, R., Wavell, S., Dimino, J., & Gersten, R. (2022). *Improving student mathematics performance in three distinct middle schools: Findings from a model demonstration grant*. Los Alamitos, CA: Instructional Research Group.

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Author Note

The authors would like to thank Amber Wang and Velette Bozeman for analyzing the data; and Jennifer Wong and Christopher Tran for assisting with the research and preparation of the report.

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Introduction

Data from the 2019 and 2022 National Assessment of Education Progress (NAEP) suggest that students performing below grade level in elementary school continue to fall further behind their peers in middle school, widening the achievement gap in mathematics for these students (NAEP, 2019, 2022). In 2019, those who performed at the 10th percentile demonstrated significantly weaker performance than a decade ago and the performance of students at the 25th percentile remained stagnant. The 2022 data show learning loss for all students, no doubt due to the COVID-19 pandemic, and still demonstrate a widening gap between lower- and higher-performing students from 4th to 8th grade. These data demonstrate that mathematics intervention in middle schools, in its current form, is not working to boost outcomes for students with mathematics learning disabilities (MLD) or mathematics difficulties (MD). Addressing this achievement gap and helping students prepare for algebra, a critical gateway to higher-level high school mathematical courses, graduation, college entrance, and success in many high-paying vocations (National Mathematics Advisory Panel [NMAP], 2008; Vogel, 2008), is a priority.

Misunderstanding abstract concepts associated with rational number sense has been shown as one of the critical areas of mathematics responsible for the achievement gap for students with MLD (Torbeyns et al., 2015). In research by Mazzocco and Devlin (2008), findings indicated that 8th-graders with MLD performed significantly worse than their peers on rational number sense. They were far less accurate in (a) naming decimal fractions, (b) knowing which fractions were equivalent (e.g., were unable to indicate whether .75 and $\frac{3}{4}$ are equivalent), and (c) ordering fractions and/or decimals by magnitude. They also tended to make little progress in this domain during the course of middle school.

The importance of this deep understanding of rational numbers and why it is crucial for success in algebra was highlighted in the report by the NMAP (2008), *Foundations for Success.* Subsequent studies (Geary et al., 2012; Siegler et al., 2012) have demonstrated that performance on rational number topics strongly predicts success in algebra.

Improved algebraic reasoning—which requires a strong understanding of rational numbers—is another area that is critical for success in algebra. Algebraic reasoning has been defined as, "forming generalizations from experiences with number and computation, formalizing these ideas with the use of a meaningful symbol system, and exploring the concepts of pattern and function" (Van De Walle et al., 2013, p. 258). It includes the ability to generalize arithmetic; analyze quantitative relationships; model, justify or prove; make predictions; solve word problems; and understand the underlying structure of problems (Cai & Knuth 2005; Kieran, 2004).

There are two critical areas of misunderstandings related to algebraic reasoning (Berch, 2005). The first is understanding the *meaning of the equal sign*, (Alibali et al., 2007), which is understanding that the equal sign represents equivalence and denotes numerical relationships. The second is the *meaning of a variable,* which is perhaps the most critical concept in algebra and is important for solving unknowns and making predictions (Alibali et al., 2014). Even though many aspects of mathematics are important for algebraic reasoning, these two big ideas are deemed the most crucial, as they are necessary for solving algebraic equations and generating equations when solving word problems.

Therefore, reducing the mathematics achievement gap in middle school requires a targeted focus on the development of algebraic reasoning for students with MLD or MD. Preparing students to take and pass algebra by 9th grade provides students the opportunity to take advanced mathematics courses throughout high school (Spielhagen, 2006) in anticipation of college requirements and entering the 21st century workforce.

The primary goal of this implementation project was to examine the feasibility of a job-embedded professional development (JEPD) for improving mathematics outcomes for students with MLD or MD at the middle school level. Our focus was on addressing grade-level content as well as foundational concepts covered in earlier grades essential for better mathematics outcomes at the middle school level and preparing students for success in algebra in the high school.

Job-embedded Professional Development

The conceptual framework of the JEPD was based on two hypotheses. First, we hypothesized that students would benefit from receiving mathematics intervention that is grounded in the best available evidence from controlled research. Second, we hypothesized that interventions for students in middle school focus on the development of rational number sense and algebraic reasoning to prepare students for advanced mathematics in high school.

The job-embedded professional development (JEPD) was designed to include the best practices for affecting change in teaching practice. These include active learning, coherence with existing practices; sufficient duration and intensity, and working collaboratively with relevant colleagues (Desimone, 2009; Garet et al., 2001).

The JEPD focused on three key features: (a) Use of evidence-based curriculum for teaching students, (b) Use of evidence-based instructional practices, and (c) Use of coaching to respond to the needs of teachers and students. The JEPD was designed to work flexibly within a variety of school contexts so that it could be adjusted based teachers' needs and strengths.

Evidence-based Intervention Curricula

We hypothesized that a solid evidence-based curricula would provide the necessary anchor for delivering the JEPD. We identified two evidence-based mathematics curricula, *TransMath* (3rd edition; Woodward & Stroh, 2015) and *Fraction Face-Off!* (Fuchs et al., 2015), that could serve as an intervention platform for building grade level mathematics as well as the missing foundational skills. Both these curricula have been designed to incorporate several evidence-based practices such as explicit instruction, visual representations, use of multiple strategies, cumulative review, and careful selection and sequencing of examples.

We wanted to use *TransMath* as the primary intervention curriculum. We anticipated using *Fraction Face-Off!* in Tier 3 to address the needs of those students who need more focused intensive instruction in fractions. Both of these interventions meet the criteria articulated in ESSA's *Non-Regulatory Guidance: Using Evidence to Strengthen Education Investments* for evidence-based practices—that is, there is at least one rigorous RCT demonstrating significant positive impacts on relevant aspects of mathematics performance. The two curricula and the evidence base are described below.

TransMath. *TransMath* (3rd edition) is a grade 5-8 intervention curriculum that focuses on the concepts needed for successful entry into algebra. It includes well-designed lessons, exemplified with effective instructional practices for teaching middle school mathematics content, as well as for building conceptual understanding and algebraic reasoning. Units and lessons from Level 3 specifically focus on middle school mathematics curriculum, and address foundational understandings for algebraic reasoning, which include: rational number concepts (e.g., fractions, decimals, and percentages); variables; ratios and proportions; inequalities; and algebraic properties and expressions of relationships. Lessons focus first on building conceptual foundations before solving procedural calculations. This design is aligned with the current ideas for best practice that balance conceptual understanding and procedural fluency (Heitin, 2014). The lessons are structured to offer sufficient time on concrete examples, which then lead to abstract notation and problem solving.

TransMath is not scripted and offers detailed lessons. While *TransMath* was developed for large-group implementation, it moves at a slower pace than a typical mathematics curriculum. Concrete and semi-concrete visual representations are woven throughout *TransMath* and mathematics vocabulary is introduced and used in subsequent lessons. *TransMath* is also designed to systematically build student learning relying on explicit instruction, as previously described.

Evidence base for the TransMath intervention. The evidence base for the *TransMath* intervention is summarized in Table 1. The evidence includes three studies: (a) an RCT examining the impact of *TransMath* fraction curriculum on 5th-grade students conceptual and procedural understanding; (b) a QED examining *TransMath* curriculum on the mathematics outcomes of Grade 6 students; and (c) a QED examining the impact of *TransMath* on middle school students.

Table 1. Studies Examining the Impact of* TransMath *Intervention

Authors	Sample	Design	Measures	Effect size
Instructional Research Group (2017)	184 5th-grade students between the 15th and the 35th percentiles	RCT	1. *Test of Understanding Fractions, Fourth Grade* 2. *Test of Understanding Fractions, Fifth Grade* 3. *Test of Fractions Procedures* 4. *Number Line Estimation 0-1* 5. *Number Line Estimation 0-2*	1. g = 0.8134*** 2. g = 0.7023*** 3. g = 1.0878*** 4. g = 1.0730*** 5. g = 0.8614***
Woodward and Brown (2006)	53 6th-grade students	QED	1. *CTB TerraNova* 2. *Core Concepts Test*	1. d = 1.23* 2. d = 1.61***
Voyager Learning (n.d.)	79 middle school students	QED	1. *Florida Comprehensive Assessment Test (FCAT)*	1. Year 2: $F(1,78) = 145.20^{a***}$

* p=.05; *** p=.001.

[a] *Effect sizes were not given for the Voyager Learning (n.d.) study.*

Fractions Face-Off! addresses essential rational number knowledge for fractions, but not decimals or percentages. It is a scripted program developed for mathematics intervention settings focused on material covered in standards from Grades 3, 4, and 5. The program is intended to be implemented in small groups to students with MD and MLD. Fractions Face-Off! builds learning in small increments so that students have sufficient time to apply learning while concepts become increasingly complex. It was designed with explicit, systematic instruction at the forefront and connects visual representations, both concrete and semi-concrete, to mathematical notation. Throughout t is an emphasis on teacher and student use of precise mathematical language.

Evidence base for the Fraction Face-Off! intervention. The evidence base for the *Fraction Face-Off!* intervention is summarized in Table 2. The evidence base includes a series of five replicated studies. Data reported below are from three publications: (a) an RCT examining the impact of *Fraction Face-Off!* on low-performing

4th-grade students (below the 35th percentile) randomly assigned to 2 treatment conditions (fluency or conceptual) or control; (b) data from the first three studies in the series analyzed for students below the 10th percentile; and (c) data from five replications to examine impacts of *Fraction Face-Off!*.

Table 2. Effects of* Fraction Face-Off! *Intervention

Study	Sample	Design	Measures	Effect size
Fuchs et al. (2014)	243 4th- grade students (84 in the Fluency group, 79 in the Conceptual group)	RCT[a]	1. *Fraction Number Line* 2. *National Assessment of Educational Progress (NAEP)–selected items* 3. *Fraction Battery 2011–Revised*	Fluency: 1. ES = 0.98** 2. ES = 0.63** 3. ES = 1.44** Conceptual: 1. ES = 1.22** 2. ES = 0.64** 3. ES = 1.41**
Fuchs et al. (2015)	203 4th-grade students	RCT[a]	1. *Comparing Fractions from the Fraction Battery–2012–Revised* 2. *Fraction Addition and Subtraction* 3. *NAEP items*	Average ES across the 3 years: 1. ES = 1.21** 2. ES = 1.71** 3. ES = 1.11**
Fuchs et al. (2016)	Approx. 250 4th-grade students per year	RCT	1. *Number line estimation* 2. *Fraction Addition and Subtraction* 3. *19 released items from 1990-2009 National Assessment of Educational Progress (NAEP)*	Average ES[b] across the 5 years: 1. $g = 0.93$ 2. $g = 1.72$ 3. $g = 0.58$

** p=.01.
[a] *A review conducted by the WWC indicates that the study meets standards without reservations.* [b]*p-values were not provided for the Fuchs, Malone, Schumacher, Namkung, and Wang (2016) study that includes all five years.*

Evidence-Based Practices

The JEPD focused on three key instructional practices: use of explicit, systematic instruction, use of visual representations, and teaching precise mathematical language. These practices are recommended in the most recent practice guide developed at IES that rigorously reviewed the mathematics intervention literature (Fuchs et al., 2021). These practices also appear in other meta-analyses focused on mathematics intervention (Gersten et al., 2008; Dennis et al., 2016) and are evident in many effective programs.

***Explicit, systematic instruction*.** The National Mathematics Advisory Panel (NMAP, 2008) report recommended "that students with learning disabilities and students with learning problems receive, on a regular basis, some explicit, systematic instruction…" (p. 48). Facets of explicit instruction include: (a) high levels of teacher-student interaction; (b) clear teacher explanations and demonstrations of mathematical concepts that enhance student understanding by modeling their thinking process; (c) positive, clear, and constructive feedback for correct and incorrect responses; and (d) ample opportunities for students to practice what they learn. (NMAP, 2008; Star et al., 2015). Explicit instruction in mathematics entails providing clear models of strategies for approaching a problem with discussions to clarify why a given approach works. This should be done consistently as students use and adapt new strategies.

***Visual representations*.** Concrete manipulatives such as Pattern Blocks, Cuisenaire Rods, and Fraction Tiles are widely used in the elementary grades, but rarely in middle school. Yet the approach has been consistently linked to better outcomes for students with MLD (Fuchs et al., 2021), especially for those with weak working memory (Fuchs et al., 2014). Thus, one goal of the JEPD was to ensure that teachers understood how to use a mix of concrete, three-dimensional manipulatives and semi-concrete, two-dimensions visual representations with knowledge on how to connect these representations to each other and with numerical representations.

***Teaching precise mathematical language*.** By using informal, "cute" language such as "railroad track" for equal sign, or "first floor" for the denominator, students lose access to what the mathematical terms such as *equivalent* and *denominator* mean. An emphasis in the JEPD was helping teachers use precise mathematical language and terminology consistently so that it becomes a typical part of their instruction. Through modeling and using precise language, teachers can extend student learning by supporting students in their use of precise language when explaining their mathematical reasoning. Teacher support may be in the form of asking questions or providing prompts to help students start an explanation. Another tactic may be to have teachers build off of students' explanations to foster a deeper understanding of the concept. Using students' explanations as examples for other students in class also helps foster discussion and explanations. Verbalizing problem-solving decisions and the steps students make while solving a problem may also facilitate self-regulation for students with MLD (Gersten et al., 2009).

Coaching

At each site coaching and instructional supports were provided based on the needs of the teachers and their students. To accomplish this, we conducted observations, met with teachers, and provided coaching and consultation to respond directly to teachers' strengths and areas of growth. We structured coaching sessions to empower teachers to problem solve alongside coaches. We also used coaching as a formative assessment for the research team so that we could design and plan future professional development sessions according to observed challenges and teacher requests.

Method

The implementation study was conducted in three school districts—two from the southwest region and one from the southeast region of the United States. Three schools, one from each school district, participated in the study. Of these three schools, two were middle schools serving students in Grades 6–8, and one was an elementary school for students in Grades K–6, where we focused on Grades 5 and 6. In this paper, the three schools are referred to as School 1, School 2, and School 3. Implementation occurred across a three-year period (2018–2021), with School 1 during the 2018–2019 academic year, School 2 during 2019–2020, and School 3 during 2020–2021.

School 1 (2018–2019)

Setting and Service Delivery Model

Students with MLD or MD performing below grade level received their mathematics instruction in a Basic Mathematics class. Students did not receive any additional intervention in mathematics. These 45-minute Basic Mathematics classes were co-taught by a pair of teachers—a general education mathematics teacher and a special education teacher.

The co-teachers used the general education core mathematics standards guide instruction and supplemented it with material from other online sources (e.g., Pinterest or Teachers Pay Teachers) to address the gap in students' foundational skills. Their stated goal was to prepare students for the Algebra 1 course in high school that is necessary for high school graduation.

Teacher and Student Participants

Three teachers, one general education mathematics teacher and two special education teachers, participated in the study. One special education teacher co-taught Grade 6 and Grade 7 Basic Mathematics classes with the general education mathematics teacher. The second special education teacher co-taught Grade 8 Basic Mathematics class along with the general education mathematics teacher. See Table 3 for each teacher's education level, certification, and years of teaching experience.

Table 3. Teaching Experience and Education Levels in School 1

	T1	T2	T3
Degree	Master's	Master's	Master's
Certification	Sp.Ed., Admin	Sp.Ed.	Secondary Math
Teaching experience in years	9	6	9
Teaching in current school in years	2	1	0
Teaching mathematics in middle school in years	2	3	3

Note. T = Teacher. Sp.Ed. = Special Education.

Forty-four students (16 Grade 6, 14 Grade 7, and 14 Grade 8) with MLD or MD from these three self-contained classrooms participated in the study. Of these 44 students, 38 were students with MLD, two had 504 plans, and four were students with MD. Compared to normative data, students' average scores on the fall 2018 *Star Math* assessment (Renaissance, 2018) were typical of students in Grades 3–4 (i.e., 3–4 grade levels below their assigned grade). The entry-level grade equivalents based on the *Star Math* assessment are presented in Table 4. As evident from the table, each Basic Mathematics class included a heterogeneous group of students.

Table 4. Grade Level Equivalencies Based on Star Math in School 1

	School 1 (2018-2019)	
	Mean	Range
Students with or at-risk for math disabilities		
Grade 5	N/A	N/A
Grade 6	3.6	1.1–6.6
Grade 7	4.3	2.6–6.6
Grade 8	4.1	2.5–5.6

	School 1 (2018-2019)	
	Mean	Range
Reference sample of typically achieving students		
Grade 6	7.9	5.3–9.0
Grade 7	6.2	5.3–7.6
Grade 8	9.9	7.4–11.0

Note. Reference sample = typically achieving students.

In addition, a random sample of typically achieving students from Grades 6, 7, and 8 general mathematics classrooms was selected to provide a reference point for examining the progress made by students with MLD or MD. In this paper, this set of typically achieving students is being referred to as the reference sample, which included 31 students from 20 core mathematics classrooms taught by 7 general education teachers (19 in Grade 6, five in Grade 7, and seven in Grade 8). See Table 4 for entry-level grade equivalents based on *Star Math* at the start of the study.

Implementation of Site-Specific Job-Embedded Professional Development

As noted earlier, our approach for providing JEPD at each site focused on three core features. One feature focused on the systematic use of an evidence-based curriculum, which formed the foundation upon which all other supports were provided. Two curricula were offered: *TransMath* along with VMathLive (supplemental online material that directly corresponds with the *TransMath* lessons) and *Fraction Face-Off!*. The second feature addressed the use of evidence-based practices. Specific guidance was provided on incorporating three evidence-based practices into the instructional routine: making instruction explicit, using visual representations to clarify concepts and model the problems, and facilitating students' explanations that highlight their understanding. The third feature, responsive coaching, focused on site-specific needs.

Implementation at School 1 occurred pre-COVID-19, and implementation of job-embedded PD was not impacted. Implementation of JEPD began in the summer preceding the school year and consisted of 25.5 hours of PD sessions delivered in the following manner: one full day in the summer, one full day just before the start of the school year, one half day during the fall semester, and one half day and one full day during the winter semester. A coaching session was held once a month with each teacher and included feedback on the lesson that was observed. Other guidance and supports based on teacher requests and our own observations included, but were not limited to, guidance on grouping children, providing differentiated instruction, and encouraging student participation and explanations, as well as developing supplemental instructional material. As the school did not provide common planning time for the co-teachers to plan, four after-school one-hour sessions were conducted. The co-teachers used this time to work together to plan upcoming lessons. For additional details on the content and focus of the JEPD, see Table 5.

Table 5. Implementation of Job-Embedded PD at School 1

Features of Job-Embedded PD	School 1 (2018-2019)
Mathematics content focus	• Rational numbers • Algebraic reasoning
Number of PD sessions	5
Number of hours of PD	25.5
Number of observations per teacher	8
Number of coaching feedback sessions per teacher	8
Lesson planning sessions	4
Informal/virtual check-ins	6
Feature 1: Provide access to evidence-based curriculum	• *TransMath* • VMathLive (online supplemental material for *TransMath*)
Feature 2: Provide guidance on evidence-based practices	• Making instruction explicit • Using visual representations (number line, Cuisenaire rods, algebra tiles) • Use of precise mathematical language

Features of Job-Embedded PD	School 1 (2018-2019)
	• Facilitating student explanations (routine use of think alouds
Feature 3: Responsive Coaching based on site-specific needs	• Guidance on using the curricula • Planning lessons • Shared student *Star Math* data after fall and winter testing • Using *Star Math* data to sort students into small groups • Providing supplemental instructional materials (e.g., worked examples and sample problems) • Providing strategies for differentiated instruction and engaging students • Providing time for planning along with teachers during PD sessions • Providing monthly coaching based on classroom observations • Providing weekly after-school planning time[a]

[a] Provided in the last two months of the school year for the co-teachers to meet and plan lessons jointly.

School 2 (2019–2020)

Setting and Service Delivery Model

The service delivery model in School 2 was completely different from the one in School 1. Students with MLD or MD received grade-level mathematics instruction in a general mathematics classroom for 70 minutes each day. Students were not assigned to classes based on prior mathematics achievement or disability status, as was done in School 1.

Students with MLD received additional mathematics instruction in a resource room for an extra 70 minutes on alternate days. This additional time was used mainly to provide support on completing classwork or homework assigned in their general mathematics class. Students received support individually, in small groups, or from computer-based, online instruction.

Students with MD (i.e., those who were not performing at grade level and were referred to intervention based on *Star Math* benchmark data) received additional targeted mathematics intervention on alternate days for 70 minutes. Learning was monitored using the *EasyCBM* measure (University of Oregon, 2006), which was already in place at the school. The intervention teacher supported students by addressing specific gaps in their learning as indicated by the *EasyCBM* data. The goal was to remediate these gaps and then release students from receiving additional mathematics intervention.

Teacher and Student Participants

Three teachers participated in the study from School 2. One teacher taught four intervention classes. The other two teachers taught special education classes: one taught a special education mathematics class for Grade 7 students, and the other taught two special education mathematics classes for students in Grades 6 and 8, respectively. Teachers from the general mathematics class were not a part of the study. See Table 6 for each teacher's education level, certification, and years of teaching experience.

Table 6.Teaching Experience and Education Levels in School 2

	T4	T5	T6
Degree	Bachelor's	Master's	Master's
Certification	K-12	K-6	Sp.Ed.
Teaching experience in years	15	6	2
Teaching in current school in years	3	2	2
Teaching mathematics in middle school in years	15	2	2

Note. T = Teacher. Sp.Ed. = Special Education.

Forty-two students (16 in Grade 6, 15 in Grade 7, and 11 in Grade 8) with MLD or MD from these seven classrooms participated in the study. Of these 42 students, 22 were students with MLD, two had 504 plans, and 18 were students with MD. Compared to normative data, students' average scores on the fall 2018 *Star Math* assessment were typical of students in Grade 4 (i.e., 2–4 grade levels below their assigned grade). See Table 7.

Table 7. Grade Level Equivalencies Based on Star Math in School 2

	School 2 (2019-2020)	
	Mean	Range
Students with or at-risk for math disabilities		
Grade 5	N/A	N/A
Grade 6	4.0	2.5–5.6
Grade 7	4.5	2.3–6.3
Grade 8	4.6	2.7–6.7
Reference sample of typically achieving students		
Grade 6	8.1	4.7–9.0
Grade 7	8.2	4.0–10.0
Grade 8	9.8	5.9–11.0

Note. Reference sample = typically achieving students.

As in School 1, a random sample of typically achieving students was selected to provide a reference point. A reference sample of 94 typically achieving students was drawn from the classrooms of three general education core mathematics teachers (33 in Grade 6, 42 in Grade 7, and 19 in Grade 8). See Table 7.

Implementation of Site-Specific Job-Embedded Professional Development

Implementation at School 2 was heavily impacted by a tornado that damaged the building as well as the COVID-19 restrictions. Our ability to provide JEPD was disrupted in March 2020, when COVID-19 restrictions were being put into place. The school did not open the rest of the school year in any formal way; students remained at home, and teachers provided check-ins via Zoom, without teaching new content or requiring any schoolwork.

At School 2, one full day of PD was provided just before the start of the school year, followed by three PD sessions during the school year—two in the fall (one full day and one half day) and one full day in winter. As in School 1, a brief coaching session was held once a month to provide feedback on the lesson observed. We conducted lesson demonstrations and assisted with the selection and organization of the *TransMath* lessons, in response to the teachers requesting help with selecting, planning, and implementing the lessons. See Table 8.

Table 8. Implementation of Job-Embedded PD at School 2

Features of Job-Embedded PD	School 2 (2019-2020)
Mathematics content focus	• Rational numbers • Algebraic reasoning
Number of PD sessions	4
Number of hours of PD	25
Number of observations per teacher	6
Number of coaching feedback sessions per teacher	2
Lesson planning sessions	2
Informal/virtual check-ins	2
Feature 1: Provide access to evidence-based curriculum	• *TransMath* • VMathLive (online supplemental material for *TransMath*) • *Fraction Face-Off!*
Feature 2: Provide guidance on evidence-based practices	• Making instruction explicit • Using visual representations (number line, Cuisenaire rods, algebra tiles) • Use of precise mathematical language
Feature 3: Responsive Coaching based on site-specific needs	• Guidance on using the curricula including lesson demonstrations • Providing guidance on lesson planning • Providing monthly coaching based on classroom observations • Sharing student *Star Math* data after fall and winter testing • Providing time for planning along with teachers during PD sessions

[a] Provided in the last two months of the school year for the co-teachers to meet and plan lessons jointly.

School 3 (2020–2021)

Setting and Service Delivery Model

Students with MLD received core mathematics instruction from a general education teacher for 30 minutes. In addition, they were pulled out of the classroom and into a learning center for an additional 25-minute small-group intervention in mathematics. Due to COVID-19 restrictions, students attended the core mathematics class and the mathematics intervention either in person or virtually.

Teacher and Student Participants

Two special education teachers participated in the study. The two teachers taught the 25-minute small-group intervention classes. See Table 9 for each teacher's education level, certification, and years of teaching experience.

Table 9. Teaching Experience and Education Levels in School 3

	T7	T8
Degree	Master's	Master's
Certification	Sp.Ed., Admin	K-8, Multiple Subject (K-12), Sp.Ed.
Teaching experience in years	5	43
Teaching in current school in years	1	Not available
Teaching mathematics in middle school in years	1	43

Note. T = Teacher. Sp.Ed. = Special Education.

Eleven students with MLD participated in the study (6 in Grade 5, and 5 in Grade 6). Compared to normative data, students' average scores on the fall 2018 *Star Math* assessment were typical of students in Grade 3 (i.e., 2–3 grade levels below their assigned grade). See Table 10. We did not have access to a reference sample at School 3 due to school-based decisions regarding COVID-19 restrictions.

Table 10. Grade Level Equivalencies Based on Star Math in School 3

	School 3 (2020-2021)	
	Mean	Range
Students with or at-risk for math disabilities		
Grade 5	3.4	2.3–4.5
Grade 6	3.6	1.8–4.9
Grade 7	N/A	N/A
Grade 8	N/A	N/A
Reference sample of typically achieving students		
Grade 6	N/A	N/A

Grade 7	N/A	N/A
Grade 8	N/A	N/A

Note. Reference sample = typically achieving students.

Implementation of Site-Specific Job-Embedded Professional Development

Implementation at School 3 was also heavily impacted by COVID-19 restrictions. The school utilized a hybrid learning model, with students attending classes either virtually or in person. The teachers were interested mainly in implementing an evidence-based curriculum (*TransMath*) as part of their work with us. A two-hour PD session was provided on the *TransMath* curriculum, which was used during small-group intervention classes. Additional guidance on any teacher-requested topics was provided via Zoom check-in calls. See Table 11.

Table 11. Implementation of Job-Embedded PD at School 3

Features of Job-Embedded PD	School 3 (2020-2021)
Mathematics content focus	• Rational numbers
Number of PD sessions	1
Number of hours of PD	2
Number of observations per teacher	4
Number of coaching feedback sessions per teacher	0
Lesson planning sessions	0
Informal/virtual check-ins	4
Feature 1: Provide access to evidence-based curriculum	• *TransMath* • VMathLive (online supplemental material for *TransMath*)
Feature 2: Provide guidance on evidence-based practices	• Making instruction explicit • Using visual representation (number line, Cuisenaire rods)
Feature 3: Responsive Coaching based on site-specific needs	• Guidance on using the curricula • Discussing student *Star Math* data after winter testing • Using *Star Math* data to sort students into classes • Providing strategies for engaging students

[a] Provided in the last two months of the school year for the co-teachers to meet and plan lessons jointly.

Measures of Student Mathematics Performance Used in all Three Sites

Star Math

Star Math is a computer-adaptive assessment of mathematics knowledge (Renaissance Learning, 2017). It provides information on mathematics performance across multiple domains for students in Grades 1–12 (e.g., whole number operations; fractions, ratios, proportions, and percentages; expressions and equations; functions; geometry; algebra; and statistics). Internal consistency estimates range from 0.93 to 0.94 for Grades 6–8 (Renaissance Learning, 2018). Alternate form reliability for Grades 6–8 is 0.84. The median concurrent validity estimates for Grades 6–8 range from 0.72 to 0.74. Median predictive validity estimates across three state assessments for Grades 6–8 range from 0.74 to 0.75. The measure was administered to all participants in the fall, winter, and spring of the school year at School 1 and School 3, but only in the fall and winter at School 2.

Star Algebra

Star Algebra is a computer-adaptive assessment that provides information on student achievement in the domain of algebra (e.g., algebra; functions; geometry; and number and quantity). Internal consistency estimates range from 0.89 to 0.91 for Grades 9–12 (Renaissance Learning, 2021; seventh- and eighth-grade scores were not used by Renaissance in the calculation of reliability estimates). Split-half reliability estimates for Grades 9–12 also ranged from 0.89 to 0.91. Test-retest reliability estimates for Grades 9–12 range from 0.71 to 0.74. The measure was administered to Grade 7 and 8 participants in the fall, winter, and spring of the school year at School 1 and only in the fall and winter at School 2. *Star Algebra* was not administered in School 2 as it is not available for Grade 6 students.

Measures of Implementation Used in all Three Sites

Teacher Appraisal Survey

A Teacher Appraisal Survey was used to gather information on the perceived usefulness and benefits of the job-embedded PD. The items addressed the helpfulness of the components of the job-embedded PD (e.g., coaching, curriculum, evidence-based instructional practices, and supports), the level of implementation, and the likelihood of continued use. Items were rated using a 4-point interval scale. The Appraisal Survey was administered to all teacher participants in late spring of the school year.

Levels of Use *Interview System*

The *Levels of Use* (LoU) interview system, developed by Hall and Hord (1988), was used to understand current implementation levels of the job-embedded PD. Each teacher was interviewed individually using the LoU interview protocol in the late spring of the school year. Each LoU interview lasted between 15 and 60 minutes. All interviews were transcribed and rated independently by two study authors. Based on teacher responses, an implementation level was assigned for each teacher: 0 = Non-use, 1 = Orientation, 2 = Preparation, 3 = Mechanical Use, 4A = Routine, 4B = Refinement, 5 = Integration, and 6 = Renewal. All disagreements were discussed by the two raters, and a consensus was reached to obtain final levels for each teacher. Rater agreement within 1, which is standard for assessing reliability on this measure, was 100%. See Supplemental Material, Appendix A for additional information on the levels.

Stages of Concern *Questionnaire*

The *Stages of Concern* (SoC) questionnaire developed by Hall and Hord (1988) was used to determine each teacher's affective reactions to the expectations posed by the JEPD after one year of implementation. The SoC is a 35-item questionnaire that assesses teachers' commonly experienced concerns when developing expertise with a curriculum or instructional practices. The test-retest reliability for the SoC ranges from 0.65 to 0.86; internal consistency estimates range from 0.64 to 0.86 (Hall & Hord, 2001).

The SoC survey was administered to all teacher participants in the late spring of the school year. Responses to the SoC survey were scored using established procedures from Hall and Hord (2001) and George et al. (2006). Each teacher's responses were categorized into seven stages of concern, with scores ranging from 0–6. The stages of concern were further divided into 3 main categories: self-concerns (Stages 0–2), task concerns (Stage 3), and impact concerns (Stages 4–6). See Supplemental Material, Appendix B for additional information on stages.

Results

Student Measures

At all three schools, the target sample grew from pretest to posttest[1] on the two benchmark adaptive mathematics assessments—*Star Math* and *Star Algebra*. For the two sites with a reference sample—School 1 and School 2—the mathematics performance growth rate of the target sample was not significantly different from the reference sample, on both the *Star Math* and *Star Algebra* assessments. The target sample consisted of students with MLD and MD and the reference sample consisted of typically achieving students in the school.

Even though the target students grew at a rate that was comparable to that of their typically achieving peers, they did not make sufficient growth to close the

[1] Fall to Spring for Dwyer and Luther; Fall to Winter for Wilson due to COVID-19 lockdowns.

achievement gap. See Tables 12, 13, and 14 for mean scaled scores, grade equivalent scores, and percentile rankings for the target and reference samples from School 1, School 2, and School 3, respectively.

Table 12. Student Performance on Star Math *and* Star Algebra *Assessments in School 1*

			Fall Pretest			Spring Posttest			Pre-Post Gain	Difference Between Groups			
	n	Grade	Mean scaled scores (SD)	GE	Percentile	Mean scaled scores (SD)	GE	Percentile	Mean scaled score (SD)	Mean scaled scores	*t (df)*	*g*	*p*-value
Star Math													
Students with MLD	44	6, 7, 8	578.23	NA	NA	595.70	NA	NA	17.48	Group 1 versus Group 13			
or MD (Group 1)			(95.13)			(107.17)			(101.54)	-1.78	-0.09 (73)	-0.02	0.938
Students with MLD	16	6	546.94	3.4	5th	560.50	3.6	7th	13.56				
or MD (Group 2)			(111.95)			(105.80)			(142.21)				
Students with MLD	14	7	601.64	4.1	8th	645.79	4.7	15th	44.14				
or MD (Group 3)			(83.87)			(66.12)			(77.10)				
Students with MLD	14	8	590.57	4	6th	585.86	3.9	5th	-4.71				
or MD (Group 4)			(80.49)			(127.95)			(59.24)				
Students with MLD	38	6, 7, 8	575.87	NA	NA	592.26	NA	NA	16.39	Group 5 versus Group 13			
(Group 5)			(96.10)			(111.79)			(109.08)	-2.86	-0.14 (67)	-0.03	0.893
Students with MLD	13	6	549.23	3.5	5th	558.92	3.6	6th					
(Group 6)			(119.68)			(115.21)							
Students with MLD	13	7	594.31	4	7th	642.00	4.6	15th					
(Group 7)			(82.48)			(67.22)							
Students with MLD	12	8	584.75	3.9	5th	574.50	3.8	4th					
(Group 8)			(81.78)			(134.33)							
Students with MD	6	6, 7, 8	593.17	NA	NA	617.50	NA	NA	24.33	Group 9 versus Group 13			
(Group 9)			(95.81)			(75.11)			(23.67)	5.08	0.24 (35)	0.11	0.810
Students with MD	3	6	537.00	3.3	4th	567.33	3.7	7th					
(Group 10)			(88.73)			(65.03)							

			Fall Pretest			Spring Posttest			Pre-Post Gain	Difference Between Groups			
	n	Grade	Mean scaled scores (SD)	GE	Percentile	Mean scaled scores (SD)	GE	Percentile	Mean scaled score (SD)	Mean scaled scores	*t (df)*	*g*	*p*-value
Students with MD (Group 11)	1	7	697.00 (--)	5.5	27th	695.00 (--)	5.5	27th					
Students with MD (Group 12)	2	8	625.50 (88.39)	4.4	9th	654.00 (59.40)	4.8	13th					
Reference sample (Group 13)	31	6, 7, 8	798.35 (66.11)	NA	NA	817.61 (58.24)	NA	NA	19.26 (49.80)				
Reference sample (Group 14)	19	6	798.16 (61.39)	8.2	79th	815.79 (61.11)	9.1	84th	17.63 (45.59)				
Reference sample (Group 15)	5	7	730.40 (38.19)	6.1	38th	802.00 (42.74)	8.4	64th	71.60 (54.07)				
Reference sample (Group 16)	7	8	847.43 (54.01)	12.3	68th	833.71 (63.40)	10.8	63rd	-13.71 (25.60)				
Star Algebra													
Students with MLD or MD (Group 17)	28	7, 8	617.79 (42.93)	NA	NA	671.21 (52.01)	NA	NA	53.43 (44.01)	Group 17 versus Group 22			
										5.26	0.32 (38)	0.11	0.752
Students with MLD or MD (Group 18)	14	7	620.29 (41.46)	NA	NA	660.07 (45.94)	NA	NA	39.79 (30.36)				
Students with MLD or MD (Group 19)	14	8	615.29 (45.77)	NA	NA	682.36 (56.93)	NA	NA	67.07 (51.96)				
Students with MLD (Group 20)	25	7, 8	615.20 (44.72)	NA	NA	670.56 (54.69)	NA	NA	55.36 (45.88)	Group 20 versus Group 22			
										7.19	0.41 (35)	0.10	0.681
Students with MD	3	7, 8	639.33	NA	NA	676.67	NA	NA	37.33				

	n	Grade	Fall Pretest: Mean scaled scores (SD)	Fall Pretest: GE	Fall Pretest: Percentile	Spring Posttest: Mean scaled scores (SD)	Spring Posttest: GE	Spring Posttest: Percentile	Pre-Post Gain: Mean scaled score (SD)	Difference Between Groups: Mean scaled scores	Difference Between Groups: t (df)	Difference Between Groups: g	Difference Between Groups: p-value
(Group 21)			(10.02)			(24.01)			(21.22)				
Reference sample (Group 22)	12	7, 8	748.25 (63.08)	NA	NA	796.42 (66.38)	NA	NA	48.17 (56.30)				
Reference sample (Group 23)	5	7	708.20 (42.82)	NA	NA	757.40 (69.87)	NA	NA	49.20 (62.44)				
Reference Sample (Group 24)	7	8	776.86 (61.49)	NA	NA	824.29 (51.48)	NA	NA	47.43 (56.65)				

Note. GE = Grade equivalent score. *df* = degree of freedom. Gr 6 = Grade 6. Gr 7 = Grade 7. Gr 8 = Grade 8. *g* = *Hedges' g*. NA = Not available. MLD = students with mathematics learning disabilities. MD = Students with mathematics difficulties. Reference sample = Typically performing students from general education classrooms.

Table 13. Student Performance on* Star Math *and* Star Algebra *Assessments in School 2

	n	Grade	Fall Pretest: Mean scaled scores (SD)	Fall Pretest: GE	Fall Pretest: Percentile	Winter Posttest: Mean scaled scores (SD)	Winter Posttest: GE	Winter Posttest: Percentile	Pre-Post Gain: Mean scaled score (SD)	Difference Between Groups: Mean scaled scores	Difference Between Groups: t (df)	Difference Between Groups: g	Difference Between Groups: p-value
Star Math													
Students with MLD or MD (Group 1)	42	6, 7, 8	609.24 (83.72)			637.26 (93.56)			28.02 (58.65)	Group 1 versus Group 13: 15.83	1.77 (133)	0.33	0.079
Students with MLD or MD (Group 2)	16	6	588.56 (68.78)	3.9	10th	599.81 (76.66)	4.1	12th	11.25 (55.29)				
Students with MLD or MD (Group 3)	15	7	621.93 (93.79)	4.3	11th	651.13 (114.58)	4.8	16th	29.20 (62.75)				

			Fall Pretest			Winter Posttest			Pre-Post Gain	Difference Between Groups			
	n	Grade	Mean scaled scores (SD)	GE	Percentile	Mean scaled scores (SD)	GE	Percentile	Mean scaled score (SD)	Mean scaled scores	*t (df)*	*g*	*p*-value
Students with MLD or MD (Group 4)	11	8	622.00 (90.67)	4.4	9th	672.82 (69.06)	5.1	16th	50.82 (54.62)				
Students with MLD (Group 5)	18	6, 7, 8	549.94 (68.28)			572.06 (87.51)			22.11 (56.34)	Group 5 versus Group 13			
										9.92	0.86 (109)	0.22	0.393
Students with MLD (Group 6)	7	6	566.71 (78.56)	3.7	7th	559.00 (99.06)	3.6	6th					
Students with MLD (Group 7)	5	7	509.60 (52.52)	3	2nd	529.80 (93.93)	3.3	3rd					
Students with MLD (Group 8)	6	8	564.00 (63.08)	3.6	4th	622.50 (46.11)	4.4	9th					
Students with MD (Group 9)	24	6, 7, 8	653.71 (65.07)			686.17 (64.18)			32.46 (61.14)	Group 9 versus Group 13			
										20.26	1.89 (115)	0.43	0.061
Students with MD (Group 10)	9	6	605.56 (59.16)	4.1	13th	631.56 (32.69)	4.5	18th					
Students with MD (Group 11)	10	7	678.10 (44.06)	5.2	22nd	711.80 (65.05)	5.8	31st					
Students with MD (Group 12)	5	8	691.60 (66.90)	5.4	19th	733.20 (30.18)	6.2	30th					
Reference sample (Group 13)	93	6, 7, 8	805.01 (55.91)			817.20 (67.91)	Gr 7: 8.5		12.19 (42.46)				

	n	Grade	Fall Pretest Mean scaled scores (SD)	Fall Pretest GE	Fall Pretest Percentile	Winter Posttest Mean scaled scores (SD)	Winter Posttest GE	Winter Posttest Percentile	Pre-Post Gain Mean scaled score (SD)	Difference Between Groups Mean scaled scores	Difference Between Groups *t (df)*	Difference Between Groups *g*	Difference Between Groups *p*-value
Reference sample (Group 14)	33	6	798.91 (51.31)	8.2	79th	825.73 (59.87)	10.1	87th	26.82 (35.71)				
Reference sample (Group 15)	41	7	796.27 (59.04)	8.1	62nd	803.32 (77.02)	8.5	65th	7.05 (45.27)				
Reference sample (Group 16)	19	8	834.47 (48.89)	11	63rd	832.37 (56.60)	10.8	63rd	-2.11 (41.55)				
Star Algebra													
Students with MLD or MD (Group 17)	16	7, 8	650.44 (51.75)	NA	NA	685.88 (78.02)	NA	NA	35.44 (53.26)	Group 17 versus Group 22 22.30	1.48 (74)	0.41	0.143
Students with MLD or MD (Group 18)	14	7	643.14 (48.37)	NA	NA	676.64 (79.28)	NA	NA	33.50 (53.72)				
Students with MLD or MD (Group 19)	2	8	701.50 (61.52)	NA	NA	750.50 (6.36)	NA	NA	49.00 (67.88)				
Students with MLD (Group 20)	4	7	584.25 (23.92)	NA	NA	576.75 (72.54)	NA	NA	-7.50 (89.07)	Group 20 versus Group 22 -20.63	-0.72 (62)	-0.37	0.477
Students with MD (Group 21)	12	7, 8	672.50 (37.03)	NA	NA	722.25 (33.03)	NA	NA	49.75 (28.47)	Group 21 versus Group 22 36.62	2.29 (70)	0.72	0.025
Reference sample (Group 22	60	7, 8	779.48 (53.15)	NA	NA	792.62 (68.44)	NA	NA	13.13 (53.60)				
Reference sample (Group 23)	41	7	770.22 (51.74)	NA	NA	772.27 (67.23)	NA	NA	2.05 (57.97)				
Reference sample (Group 24)	19	8	799.47 (51.88)	NA	NA	836.53 (48.28)	NA	NA	37.05 (32.68)				

Note. GE = Grade equivalent score. Gr 6 = Grade 6. Gr 7 = Grade 7. Gr 8 = Grade 8. *g* = *Hedges' g*. NA = Not available. Due to COVID-19 related restrictions, spring data could not be collected.

Table 14. Student Performance on* Star Math *in School 3

			Fall Pretest			Spring Posttest			Pre-Post Gain
	n	Grade	Mean Scaled Score (SD)	GE	Percentile	Mean Scaled Score (SD)	GE	Percentile	Mean scaled score (SD)
Star Math									
Students with MLD	11	5, 6	541.27 (86.77)			587.73 (68.45)			46.45 (65.89)
Students with MLD	6	5	535.17 (69.78)	3.3	11th	592.50 (70.04)	4	24th	57.33 (72.43)
Students with MLD	5	6	548.60 (112.31)	3.5	5th	582.00 (74.20)	3.9	9th	33.40 (62.48)

Note. MLD = Mathematics learning disabilities. GE = Grade Equivalent score. Gr 5 = Grade 5. Gr 6 = Grade 6. Due to COVID-19, there was no reference sample.

Results on Measures of Implementation

Teacher Appraisal Survey

The survey was given to teachers at the end of the implementation year to help the research team understand how the teachers felt about the various features of the JEPD. At Schools 1 and 3, teachers ranked items similarly; however, at School 2, the special education teachers rated features of the JEPD as their least favorite and the mathematics intervention teacher ranked items as their favorite. See Table 15 for the most and least highly rated features of the JEPD.

Table 15. Feedback from Teachers on the Teacher Appraisal Survey

	School 1	School 2	School 3
Most highly rated aspects of JEPD in terms of helpfulness/ usefulness	• Use of the *TransMath* curriculum • Weekly after school lesson planning sessions • Lesson Planning support • Guidance on ranking and grouping students based on Star data	• Lesson Demonstrations • Lesson Planning support • PD on using instructional curricula (both *TransMath* and Fractions Face Off • Feedback from observations (mathematics intervention teacher) • Guidance on using visual representations and online material (mathematics intervention teacher) • Professional development devoted to instructional planning (mathematics intervention teacher) • Fractions Face-Off curriculum and the associated PD on how to use it (two special education teachers)	• Use of the *TransMath* curriculum and associated PD • Data from Star testing
Least highly rated aspects of JEPD in terms of helpfulness/ usefulness	• Coaching/ feedback focused on teacher's instruction • Data from Star Math tests	• Data from Star testing • Coaching /feedback from observations • Guidance on Cuisenaire rods (two sped teachers) • Fractions Face-Off curriculum and the associated PD on how to use it (mathematics intervention teacher)	• Online *V-Math* material • Check-in calls

Levels of Use *Interview System*

In the first year of implementation at each school, all teachers fell among three categories. Those teachers at the mechanical use level reported more on the logistical aspects of using *TransMath* than on issues related to the larger picture of their instruction. Teachers in the Routine stage were comfortable with implementation and were starting to think about the integration of *TransMath* into their teaching. Teachers in the Refinement state were fully integrating *TransMath* into their teaching and were adjusting their implementation to hopefully improve student learning. Teacher LoU levels are reported in Table 16.

Table 16. LoU Scores for Teachers at the End of Year 1

Levels of Use	School 1 (2018-2019)	School 2 (2019-2020)	School 3 (2020-2021)
0: Non-use			
1: Orientation			
2: Preparation			
3: Mechanical Use	T1	T4, T6	
4A: Routine		T5	T7
4B: Refinement	T2, T3		T8
5: Integration			
6: Renewal			

Stages of Concern *Questionnaire*

The Stages of Concern questionnaire assess whether teachers are most or least concerned about (a) their role within the implementation of the JEPD features (i.e., self concerns Stages 0–2), (b) the key tasks related to implementing aspects of the JEPD (i.e., task concerns Stage 3), or (c) the impacts on their teaching and student learning (i.e., Stages 4–6). See Tables 17, 18, and 19 for teachers at School 1, School 2, and

School 3, respectively. The largest percentile indicates that is the teacher's greatest concern; the smallest percentile is the teacher's lowest concern.

Table 17. Highest Stages of Concern in School 1

	Stage 0		Stage 1		Stage 2		Stage 3		Stage 4		Stage 5		Stage 6	
	RS	P	RS	P	RS	P	RS	P	RS	P	RS	P	RS	P
T1	10	55	21	**75**	16	59	18	69	18	24	18	40	20	65
T2	3	4	6	30	4	21	13	47	7	4	19	44	21	**69**
T3	9	48	5	27	5	25	10	34	20	30	17	36	18	**57**

Note. RS = Raw score. P = percentile.

Table 18. Highest Stages of Concern in School 2

	Stage 0		Stage 1		Stage 2		Stage 3		Stage 4		Stage 5		Stage 6	
	RS	P	RS	P	RS	P	RS	P	RS	P	RS	P	RS	P
T4	11	61	13	51	7	31	16.5	**62.5**	11	8	7	9	11	26
T5	4	7	23	**84**	12	48	12	43	19	27	25	68	23	77
T6	13	**75**	7	34	6	28	7	23	14	13	9	12	12	30

Note. RS = Raw score. P = percentile.

Table 19. Highest Stages of Concern in School 3

	Stage 0		Stage 1		Stage 2		Stage 3		Stage 4		Stage 5		Stage 6	
	RS	P	RS	P	RS	P	RS	P	RS	P	RS	P	RS	P
T7	10	55	16	**60**	3	17	3	9	7	4	7	9	16	47
T8	9	48	16	**60**	1	12	3	9	7	4	4	4	16	47

Note. RS = Raw score. P = percentile.

Summary

This implementation project focused on supporting teachers who provide mathematics instruction and/or intervention to students with MLD and MD. The research team worked with three schools, over three years. Instructional delivery differed across schools, primarily in setting and amount of time students received mathematics instruction. Given these various contexts, we provided job-embedded professional development to support each school's implementation of the three key features of the project: using evidence-based curricula, using evidence-based practices, and responsive coaching.

By focusing middle school intervention on algebraic reasoning—which requires a strong understanding of rational numbers, variables, and equivalence—students with MLD and MD made gains across the school year that were comparable to their peers. However, the gains were not enough to close the achievement gap between students with MLD and MD and students who are on grade-level. Given the three-year implementation of this project included two years of altered instruction due to COVID-19 restrictions, we are encouraged that students with MLD and MD made gains that were comparable to their peers.

We worked with eight teachers across the three schools and found variability among their implementation and how they preferred to work with the research team. From an implementation science perspective, one year of implementing a new innovation is likely insufficient to fully build capacity in teachers to sustain the practices learned from the JEPD. However, teachers indicated that they learned many new things related to teaching students with MLD and MD through the JEPD, which is encouraging.

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Appendix A

***Levels of Use* Interview System**

Level 0 – **Non-use:** The individual has little or no knowledge or involvement in the innovation and is doing nothing to become involved.

Level 1 – **Orientation:** Takes action to learn more about the innovation.

Level 2 – **Preparation:** Decides to use the innovation by establishing a time to begin.

Level 3 – **Mechanical Use:** Focuses effort on day-to-day use of the innovation with time for reflection.

Level 4 A – **Routine:** Use of the innovation is stabilized.

Level 4B – **Refinement:** Varies the use of the innovation to increase impact on the clients (students or others).

Level 5 – **Integration:** Combines efforts to use the innovation with related activities of colleagues for a collective impact.

Level 6 – **Renewal:** Reevaluates the quality of the use of the innovation and makes modifications.

Reference: (Loucks et al., 1975)

Appendix B

Stages of Concern Questionnaire

Self Concerns

Stage 0 – There are no concerns about the innovation. Individuals at this level may have heard of the program but feel it has no implications on them. Conversely, individuals who are experienced in the innovation are at Stage 0 if they do not consider the program as being new or needing extra thought or attention.

Stage 1 – Individuals are concerned about learning more about the innovation and how it compares with what they are currently using.

Stage 2 – Stakeholders express doubts about their ability to use the innovation and concerns about making mistakes.

Task Concerns

Stage 3 – This stage addresses management concerns. Concerns center around fitting the innovation into their current program and the amount of time needed to implement it.

Impact Concerns

Stage 4 (Consequence) – Teachers' concerns about the impact of the innovation on students.

Stage 5 (Collaboration) – Individuals are interested in collaborating with their colleagues to improve the results of the program.

Stage 6 (Refocusing) – This stage evolves after teachers have used the program effectively and begin to consider more powerful alternatives to the innovation.

Reference: (George et al., 2006)